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Piezoresistive Stress Sensors for Structural Analysis of Electronic Packages

Structural reliability of electronic packages has become an increasing concern for a variety of reasons including the advent of higher integrated circuit densities, power density levels, and operating temperatures. A powerful method for experimental evaluation of die stress distributions is the use of test chips incorporating integral piezoresistive sensors. In this paper, the theory of conduction in piezoresistive materials is reviewed and the basic equations applicable to the design of stress sensors on test chips are presented. General expressions are obtained for the stress-induced resistance changes which occur in arbitrarily oriented one-dimensional filamentary conductors fabricated out of crystals with cubic symmetry and diamond lattice structure. These relations are then applied to obtain basic results for stressed in-plane resistors fabricated into the surface of (100) and (111) oriented silicon wafers. Sensor rosettes developed by previous researchers for each of these wafer orientations are reviewed and more powerful rosettes are presented along with the equations needed for their successful application. In particular, a new sensor rosette fabricated on (111) silicon is presented which can measure the complete three-dimensional stress state at points on the surface of a die

Introduction

Stresses due to thermal and mechanical loadings are often produced in chips which are incorporated into electronic packages. During fabrication steps such as encapsulation and die-attachment, thermally-induced stresses are created. These occur due to nonuniform thermal expansions resulting from mismatches between the coefficients of thermal expansion of the materials comprising the package and the semiconductor die. Additional thermally-induced stresses can be produced from heat dissipated by high power density devices during operation. Finally, mechanical loadings can be transmitted to the package through contact with the printed circuit board to which it is mounted. The combination of all of the above loadings can lead to two-dimensional (biaxial) and three-dimensional (triaxial) states of stress on the surface of the die. If high-power density devices within the package are switched on and off, these stress states can be cyclic in time causing fatigue. All of these factors can lead to premature failure of the package due to such causes as fracture of the die, severing of bond connections, die attach failure, and encapsulant cracking. These reliability problems are of ever increasing concern as larger scale chips and higher temperature applications are considered.

Stress analyses of electronic packages and their components have been performed using analytical, numerical, and experimental methods. Analytical investigations have been primarily concerned with finding closed-form elasticity solutions for layered structures, while numerical studies have typically considered finite element solutions for sophisticated package geometries. Experimental approaches have included the use of test chips incorporating piezoresistive stress sensors (semiconductor strain gages), and the use of optical techniques such as holographic interferometry, moire interferometry, and photoelasticity. In this paper, the theory and design of piezoresistive stress sensors are considered in detail.

Piezoresistive stress sensors are a powerful tool for experimental structural analysis of electronic packages. They are conveniently fabricated into the surface of the die as part of the normal processing procedure. In addition, they are capable of providing nonintrusive measurements of surface stress states on a chip even within encapsulated packages. If the piezoresistive sensors are calibrated over a wide temperature range, thermally induced stresses can be measured. Finally, a full-field mapping of the stress distribution over a die's surface can be obtained using specially designed test chips which incorporate an array of sensor rosettes and multiplexing circuitry.

Prior published applications of stress sensing test chips have included sensor rosettes with two and four resistors. Two element rosettes fabricated on (100) silicon have been utilized by Spencer et al. [1981], Edwards and coworkers [1983, 1987], and Beaty et al. [1990]. The sensor rosettes utilized in these investigations are able to measure two in-plane normal stress components at points in a state of plane stress such as those on the free (unloaded) surface of the integrated circuit die. Four element rosettes containing strictly *n*-type or *p*-type resistors have been applied by Natarajan and Bhattacharyya

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[1986] using (100) silicon, and Gee et al. [1988, 1989] using (111) silicon. The sensors in these studies are able to measure all three of the in-plane stress components provided that the location of the sensor is in a state of plane stress. A four element rosette on (100) silicon containing two n-type resistors and two p-type resistors has been designed and applied by Miura and co-workers [1987, 1990]. This sensor is able to measure the three in-plane stress components and the out-of-plane normal stress component. Also, the resistance changes of the rosette elements are insensitive to the out-of-plane shearing stresses. This piezoresistive stress sensor was the first to be presented with the capability to measure four of the six unique stress components at an arbitrarily stressed point.

No general presentation of the theory and design of piezoresistive stress sensors for test chips has appeared in the literature. In addition, most prior stress chips have been designed for plane stress situations so that they cannot be applied to encapsulated packages with confidence. Finally, no sensors capable of measuring the complete three-dimensional stress state at points on the surface of a die are currently available. In this paper, the theory of conduction in piezoresistive materials is reviewed and the basic equations needed for designing test chip stress sensors are derived. Emphasis is placed on obtaining general expressions for the resistance changes experienced by arbitrarily stressed in-plane resistors fabricated on (100) or (111) silicon wafers. These are the most common orientations used in silicon integrated circuit fabrication. New sensor rosettes are presented which ease calibration considerations and permit more stress components to be measured. In particular, a new six element sensor rosette fabricated on (111) silicon is introduced which can measure the complete three-dimensional stress state at points on a die's surface.

Mathematical Theory of Piezoresistivity

Anisotropic Conduction. A basic axiom of the theory of conduction of electric charge is that the current density vector is a function of the electric field vector

$$\mathbf{J}=\mathbf{J}(\mathbf{E}) \tag{1}$$

or

$$J_i=J_i(E_1, E_2, E_3) \tag{2}$$

where J_i and E_i are the cartesian components of the current density and electric field vectors, respectively. In most solid conductors, this functional relation has been observed to be linear over a wide range of electric field magnitudes. Such conductors are referred to as ohmic materials. In an anisotropic ohmic conductor, the most general linear relationship is

$$J_i=\kappa_{ij}E_j \tag{3}$$

where κ_{ij} are the components of the conductivity tensor, and the summation convention is implied for repeated indices. This relation can be inverted to give

$$E_i=\rho_{ij}J_j \tag{4}$$

where ρ_{ij} are the components of the resistivity tensor. Using the reciprocity theorem derived by Onsager [1931a, 1931b], it is possible to show that the conductivity and resistivity tensors are symmetric

$$\kappa_{ij}=\kappa_{ji},\ \rho_{ij}=\rho_{ji} \tag{5}$$

The Piezoresistive Effect. The piezoresistive effect is a stress-induced change in the components of the resistivity tensor. It is exhibited in so-called piezoresistive materials. The first observations of this phenomenon were made by Bridgman [1922, 1925, 1932] who subjected metals to tension and hydrostatic pressure. Experimental observations of the piezoresistive effect in semiconductors (silicon and germanium) were first made by Taylor [1950], Bridgman [1951], Smith [1954], and Paul and Pearson [1955].

The piezoresistive effect can be modeled mathematically using the series expansion

$$\rho_{ij}=\rho_{ij}^{o}+\pi_{ijkl}\sigma_{kl}+\Lambda_{ijklmn}\sigma_{kl}\sigma_{mn}+\cdots \tag{6}$$

where ρ_{ij}^{o} are the resistivity components for the stress free material and π_{ijkl}, Λ_{ijklmn}, etc. are components of fourth, sixth, and higher order tensors which characterize the stress-induced resistivity change. For sufficiently small stress levels, this relation is typically truncated so that the resistivity components are linearly related to the stress components

$$\rho_{ij}=\rho_{ij}^{o}+\pi_{ijkl}\sigma_{kl} \tag{7}$$

For fixed environmental conditions (i.e. temperature), the 81 components π_{ijkl} of the fourth order piezoresisitvity tensor are constants.

From Eq. (7) it is easily seen that the stress-induced resistivity changes are given by

$$\Delta\rho_{ij}=\pi_{ijkl}\sigma_{kl} \tag{8}$$

It is also possible to model the resistivity changes in terms of the strain components using an expression such as

$$\Delta\rho_{ij}=M_{ijkl}\epsilon_{kl} \tag{9}$$

where M_{ijkl} are the components of the fourth order elastoresistivity tensor. In this paper, the stress-based formulation given in Eqs. (7, 8) is adopted.

Reduced Index Notation. The relations in Eqs. (7, 8) can be simplified by recognizing that the resistivity and stress tensors are symmetric. Since $\Delta\rho_{ij}=\Delta\rho_{ji}$, it follows that the linear dependence of resistivity component $\Delta\rho_{ij}$ on stress component σ_{kl} will be the same as the linear dependence of $\Delta\rho_{ji}$ on σ_{kl}. These conditions imply the general relation

$$\pi_{ijkl}=\pi_{jikl} \tag{10}$$

In addition, the relation $\sigma_{kl}=\sigma_{lk}$ suggests that the linear dependence of resistivity component $\Delta\rho_{ij}$ on stress component σ_{kl} will be the same as its linear dependence on stress component σ_{lk}. Therefore, it can be concluded that

$$\pi_{ijkl}=\pi_{ijlk} \tag{11}$$

Using Eqs. (5, 10, 11), the nine expressions in Eq. (7) can be simplified to the following six unique equations:

$$\begin{bmatrix}\rho_{11}\\ \rho_{22}\\ \rho_{33}\\ \rho_{13}\\ \rho_{23}\\ \rho_{12}\end{bmatrix}=\begin{bmatrix}\rho_{11}^{0}\\ \rho_{22}^{0}\\ \rho_{33}^{0}\\ \rho_{13}^{0}\\ \rho_{23}^{0}\\ \rho_{12}^{0}\end{bmatrix}+\begin{bmatrix}\pi_{1111} & \pi_{1122} & \pi_{1133} & 2\pi_{1113} & 2\pi_{1123} & 2\pi_{1112}\\ \pi_{2211} & \pi_{2222} & \pi_{2233} & 2\pi_{2213} & 2\pi_{2223} & 2\pi_{2212}\\ \pi_{3311} & \pi_{3322} & \pi_{3333} & 2\pi_{3313} & 2\pi_{3323} & 2\pi_{3312}\\ \pi_{1311} & \pi_{1322} & \pi_{1333} & 2\pi_{1313} & 2\pi_{1323} & 2\pi_{1312}\\ \pi_{2311} & \pi_{2322} & \pi_{2333} & 2\pi_{2313} & 2\pi_{2323} & 2\pi_{2312}\\ \pi_{1211} & \pi_{1222} & \pi_{1233} & 2\pi_{1213} & 2\pi_{1223} & 2\pi_{1212}\end{bmatrix}\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{13}\\ \sigma_{23}\\ \sigma_{12}\end{bmatrix} \tag{12}$$

The above relations are the most concise form for the fully expanded equations of the theory of piezoresistivity. They are not convenient in a notational sense since they cannot be expressed compactly in indicial notation. Historically, it has become a convention to reduce the complexities of the index labels through a renumbering scheme where index pairs are replaced by single indices which assume values of 1, 2, ..., 6 instead of 1, 2, 3. The following index conversions are typically used:

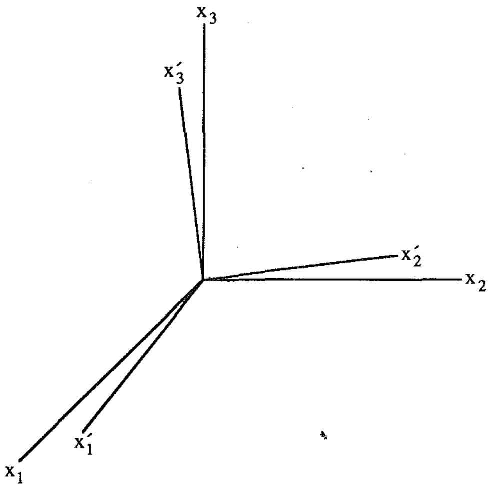

Fig. 1 Unprimed and primed coordinate systems

$$\begin{matrix} 11 \sim 1 & 22 \sim 2 & 33 \sim 3 \\ 13 \sim 4 & 23 \sim 5 & 12 \sim 6 \end{matrix} \tag{13}$$

If such a reduced index notation is adopted, the expressions in Eq. (12) can be rewritten in indicial notation as

$$\rho_\alpha = \rho_\alpha^0 + \Pi_{\alpha\beta}\sigma_\beta \tag{14}$$

where greek indices take on the values 1, 2, ..., 6 and

$$\rho_1 = \rho_{12},\ \rho_2 = \rho_{22},\ \ldots,\ \rho_6 = \rho_{12} \tag{15}$$

$$\rho_1^0 = \rho_{11}^0,\ \rho_2^0 = \rho_{22}^0,\ \ldots,\ \rho_6^0 = \rho_{12}^0 \tag{16}$$

$$\sigma_1 = \sigma_{11},\ \sigma_2 = \sigma_{22},\ \ldots,\ \sigma_6 = \sigma_{12} \tag{17}$$

$$\begin{gathered} \Pi_{11} = \pi_{1111},\ \Pi_{12} = \pi_{1122},\ \ldots,\ \Pi_{16} = 2\pi_{1112} \\ \Pi_{21} = \pi_{2211},\ \Pi_{22} = \pi_{2222},\ \ldots,\ \Pi_{26} = 2\pi_{2212} \\ \text{etc...} \end{gathered} \tag{18}$$

A further notational simplification can be obtained by introduction of the so-called piezoresistive coefficients. They are defined by

$$\pi_{\alpha\beta} = \frac{\Pi_{\alpha\beta}}{\bar{\rho}} \tag{19}$$

where $\bar{\rho}$ is the mean (hydrostatic) unstressed resistivity

$$\bar{\rho} = \frac{\rho_{11}^0 + \rho_{22}^0 + \rho_{33}^0}{3} \tag{20}$$

Substitution of Eq. (19) into Eq. (14) leads to

$$\rho_\alpha = \rho_\alpha^0 + \bar{\rho}\pi_{\alpha\beta}\sigma_\beta \tag{21}$$

or

$$\frac{\Delta\rho_\alpha}{\bar{\rho}} = \pi_{\alpha\beta}\sigma_\beta \tag{22}$$

Transformation Relations. The basic mathematical relations for conduction and piezoresistivity found in Eqs. (4, 7) are tensor equations. Thus, their form is invariant upon coordinate system transformation. A pair of cartesian coordinate systems differing in orientation is shown in Fig. 1. If Eqs. (4, 7) are valid in the unprimed system (x_1, x_2, x_3), the appropriate expressions for the primed system are

$$E_i' = \rho_{ij}' J_j' \tag{23}$$

$$\rho_{ij}' = \rho_{ij}^{0\prime} + \pi_{ijkl}'\sigma_{kl}' \tag{24}$$

The components of the electric field vector, current density vector, resistivity tensor, stress tensor, and piezoresistivity tensor all transform from one coordinate system to the other using the standard tensor transformation relations:

$$E_i' = a_{ij}E_j \qquad E_i = a_{ji}E_j' \tag{25}$$

$$J_i' = a_{ij}J_j \qquad J_i = a_{ji}J_j' \tag{26}$$

$$\rho_{ij}' = a_{ik}a_{jl}\rho_{kl} \qquad \rho_{ij} = a_{ki}a_{lj}\rho_{kl}' \tag{27}$$

$$\sigma_{ij}' = a_{ik}a_{jl}\sigma_{kl} \qquad \sigma_{ij} = a_{ki}a_{lj}\sigma_{kl}' \tag{28}$$

$$\begin{gathered} \pi_{ijkl}' = a_{im}a_{jn}a_{ko}a_{lp}\pi_{mnop} \\ \pi_{ijkl} = a_{mi}a_{nj}a_{ok}a_{pl}\pi_{mnop}' \end{gathered} \tag{29}$$

where

$$a_{ij} = \cos(x_i', x_j) \tag{30}$$

are the direction cosines for the two coordinate systems.

The mathematical models for the piezoresistive effect found in Eqs. (21, 22) were expressed in reduced index notation and are not tensor equations. However, they can be considered form invariant upon coordinate system transformation if appropriate transformation relations are introduced for the piezoresistive coefficients. This fact was first demonstrated by Smith [1958], and then later applied to crystals with cubic symmetry and diamond structure by Pfann and Thurston [1961], and Thurston [1964]. If Eqs. (21, 22) are valid in the unprimed system (x_1, x_2, x_3), the form invariant expressions for the primed system are

$$\rho_\alpha' = \rho_\alpha^{0\prime} + \bar{\rho}\pi_{\alpha\beta}'\sigma_\beta' \tag{31}$$

$$\frac{\Delta\rho_\alpha'}{\bar{\rho}} = \pi_{\alpha\beta}'\sigma_\beta' \tag{32}$$

In Eqs. (31, 32), the scalar invariant nature of the mean unstressed resistivity $\bar{\rho}$ has been recognized.

The transformation relations for the reduced index resistivity and stress components can be obtained by substituting Eqs. (15, 17) into the fully expanded versions of Eqs. (27, 28). This calculation leads to the relations

$$\rho_\alpha' = T_{\alpha\beta}\rho_\beta \qquad \rho_\alpha = T_{\alpha\beta}^{-1}\rho_\beta' \tag{33}$$

$$\sigma_\alpha' = T_{\alpha\beta}\sigma_\beta \qquad \sigma_\alpha = T_{\alpha\beta}^{-1}\sigma_\beta' \tag{34}$$

where the coefficients $T_{\alpha\beta}$ are elements of a six by six transformation matrix related by the directions cosines for the unprimed and primed coordinate systems. The specific form of this transformation matrix is

$$[T_{\alpha\beta}] = \begin{bmatrix} l_1^2 & m_1^2 & n_1^2 & 2l_1n_1 & 2m_1n_1 & 2l_1m_1 \\ l_2^2 & m_2^2 & n_2^2 & 2l_2n_2 & 2m_2n_2 & 2l_2m_2 \\ l_3^2 & m_3^2 & n_3^2 & 2l_3n_3 & 2m_3n_3 & 2l_3m_3 \\ l_1l_3 & m_1m_3 & n_1n_3 & (l_1n_3 + l_3n_1) & (m_1n_3 + m_3n_1) & (l_1m_3 + l_3m_1) \\ l_2l_3 & m_2m_3 & n_2n_3 & (l_2n_3 + l_3n_2) & (m_2n_3 + m_3n_2) & (l_2m_3 + l_3m_2) \\ l_1l_2 & m_1m_2 & n_1n_2 & (l_1n_2 + l_2n_1) & (m_1n_2 + m_2n_1) & (l_1m_2 + l_2m_1) \end{bmatrix} \tag{35}$$

where the following reduced index rotation has been introduced for the direction cosines:

$$[a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \tag{36}$$

The appropriate transformation relations for the reduced index piezoresistive coefficients are obtained by enforcing the validity of Eqs. (21, 22, 31, 32) and using the results in Eqs. (33, 34). These calculations lead to the general expression

$$\pi'_{\alpha\beta} = T_{\alpha\gamma}\pi_{\gamma\delta}T^{-1}_{\delta\beta} \tag{37}$$

Crystal Symmetry. For general anisotropic materials, the equations of conduction and piezoresistivity are very complex and contain numerous terms. However, when considering crystalline materials exhibiting lattice symmetry, several simplifications can be made. These simplifications result because relationships can be established between the components of the unstressed resistivity tensor and between the components of the piezoresistivity tensor. Detailed general expositions on the ramifications of crystal symmetry on physical properties have been presented by Nye [1957], Mason [1958], Smith [1958], Bhagavantam [1966], and Juretschke [1974].

A crystal is a solid whose local properties and structure are periodic in three dimensions. A rotation or a combination rotation/reflection of a crystal which brings its lattice structure into superposition with itself is called a symmetry operation for the crystal. The set of all symmetry operations for a given crystal defines the crystallographic point group symmetry for the crystal. All crystals with the same point group symmetry are said to be members of the same crystal class. There are 32 unique crystal classes. Silicon is a cubic crystal with diamond structure, and belongs to the crystal class denoted 32 in the international numbering system. This class has been notated several other ways including m3m and O_h.

The symmetry exhibited by a crystal determines the extent of anisotropy exhibited by the physical properties of the crystal. It is assumed that the physical properties of the crystalline material must possess at least the symmetry of the point group of the crystal. This is expressed mathematically by requiring the components of a physical property tensor for the crystal to be invariant under coordinate system transformations equivalent to the symmetry operations in the point group of the crystal. These relations hold when the initial coordinate system is aligned with the symmetry axes of the crystal. Therefore, using Eqs. (28, 29), the components of the unstressed resistivity tensor and the piezoresistivity tensor of a crystal must satisfy

$$\rho^0_{ij} = \rho^{0\prime}_{ij} = a_{ik}a_{jl}\rho^0_{kl} \tag{38}$$

$$\pi_{ijkl} = \pi'_{ijkl} = a_{im}a_{jn}a_{ko}a_{lp}\pi_{mnop} \tag{39}$$

when the direction cosines between the two coordinate systems are chosen to be equivalent to one of the crystal's symmetry operations, and the initial coordinate system is aligned with the crystal's symmetry axes. In terms of reduced index notation, these conditions take the form

$$\rho^0_{\alpha} = \rho^{0\prime}_{\alpha} = T_{\alpha\beta}\rho^0_{\beta} \tag{40}$$

$$\pi_{\alpha\beta} = \pi'_{\alpha\beta} = T_{\alpha\gamma}\pi_{\gamma\delta}T^{-1}_{\delta\beta} \tag{41}$$

The unique symmetry operations or so-called generating elements for each crystal point group have been listed by Juretschke [1974]. For a cubic crystal with diamond structure such as silicon, the coordinate system transformations equivalent to these unique symmetry operations are

$$[a_{ij}] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{42}$$

$$[a_{ij}] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{43}$$

$$[a_{ij}] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{44}$$

The simplifications in the reduced index resistivity components and the piezoresistivity coefficients of silicon required by its crystal symmetry are obtained by substituting each set of direction cosines in Eqs. (42–44) into Eqs. (40, 41). If all of these calculations are considered, the following relations are found:

$$\begin{aligned} \rho^0_1 = \rho^0_2 = \rho^0_3 = \bar{\rho} \\ \rho^0_4 = \rho^0_5 = \rho^0_6 = 0 \end{aligned} \tag{45}$$

$$[\pi_{\alpha\beta}] = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{12} & 0 & 0 & 0 \\ \pi_{12} & \pi_{11} & \pi_{12} & 0 & 0 & 0 \\ \pi_{12} & \pi_{12} & \pi_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \pi_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \pi_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \pi_{44} \end{bmatrix} \tag{46}$$

It is observed that there exists only one unique unstressed resistivity component and three unique piezoresistive coefficients. The simplified properties in Eqs. (45, 46) are for a coordinate system aligned with the principal symmetry axes of the cubic silicon crystal. In a rotated coordinate system, the resistivity components and piezoresistive coefficients of silicon can be evaluated by substituting Eqs. (45, 46) into Eqs. (33, 37). Such expressions have been evaluated in detail by Pfann and Thurston [1961], and Thurston [1964].

General Conduction Equations for Stressed Materials. The governing tensor equation of conduction in a stressed anisotropic ohmic conductor is obtained by substituting Eq. (7) into Eq. (4)

$$E_i = \rho^0_{ij}J_j + \pi_{ijkl}\sigma_{kl}J_j \tag{47}$$

Each of the three expressions ($i = 1, 2, 3$) in Eq. (47) has a right hand side containing 30 terms. This complexity can be reduced to 21 terms by rewriting the expressions in Eq. (47) using the piezoresistive coefficients and reduced index notation for the resistivities and stresses. This notational adjustment is accomplished by first substituting the relations in Eq. (15) into the expressions in Eq. (4)

$$\begin{aligned} E_1 &= \rho_1 J_1 + \rho_6 J_2 + \rho_4 J_3 \\ E_2 &= \rho_6 J_1 + \rho_2 J_2 + \rho_5 J_3 \\ E_3 &= \rho_4 J_1 + \rho_5 J_2 + \rho_3 J_3 \end{aligned} \tag{48}$$

The formulas in Eq. (21) are then substituted into these expressions to give

$$\begin{aligned} E_1 &= [\rho^0_1 + \bar{\rho}\pi_{1\alpha}\sigma_\alpha]J_1 + [\rho^0_6 + \bar{\rho}\pi_{6\alpha}\sigma_\alpha]J_2 + [\rho^0_4 + \bar{\rho}\pi_{4\alpha}\sigma_\alpha]J_3 \\ E_2 &= [\rho^0_6 + \bar{\rho}\pi_{6\alpha}\sigma_\alpha]J_1 + [\rho^0_2 + \bar{\rho}\pi_{2\alpha}\sigma_\alpha]J_2 + [\rho^0_5 + \bar{\rho}\pi_{5\alpha}\sigma_\alpha]J_3 \\ E_3 &= [\rho^0_4 + \bar{\rho}\pi_{4\alpha}\sigma_\alpha]J_1 + [\rho^0_5 + \bar{\rho}\pi_{5\alpha}\sigma_\alpha]J_2 + [\rho^0_3 + \bar{\rho}\pi_{3\alpha}\sigma_\alpha]J_3 \end{aligned} \tag{49}$$

The expressions in Eq. (49) are the most concise form for the fully expanded governing equations of conduction in a stressed anisotropic conductor. They are form invariant and are valid in all coordinate systems.

Conduction Equations for Stressed Cubic Crystals. The basic governing equations for conduction in a cubic crystal with diamond structure (e.g. silicon) under stress are obtained by substituting the simplified resistivity components and piezoresistive coefficients in Eqs. (45, 46) into the general relations given in Eq. (49). This calculation gives

$$\begin{aligned} \frac{E_1}{\bar{\rho}} &= [1 + \pi_{11}\sigma_{11} + \pi_{12}(\sigma_{22} + \sigma_{33})]J_1 + \pi_{44}\sigma_{12}J_2 + \pi_{44}\sigma_{13}J_3 \\ \frac{E_2}{\bar{\rho}} &= [1 + \pi_{11}\sigma_{22} + \pi_{12}(\sigma_{11} + \sigma_{33})]J_2 + \pi_{44}\sigma_{12}J_1 + \pi_{44}\sigma_{23}J_3 \\ \frac{E_3}{\bar{\rho}} &= [1 + \pi_{11}\sigma_{33} + \pi_{12}(\sigma_{11} + \sigma_{22})]J_3 + \pi_{44}\sigma_{13}J_1 + \pi_{44}\sigma_{23}J_2 \end{aligned} \tag{50}$$

These expressions were first presented in the literature by Mason and Thurston [1957]. They are valid only if the coordinate

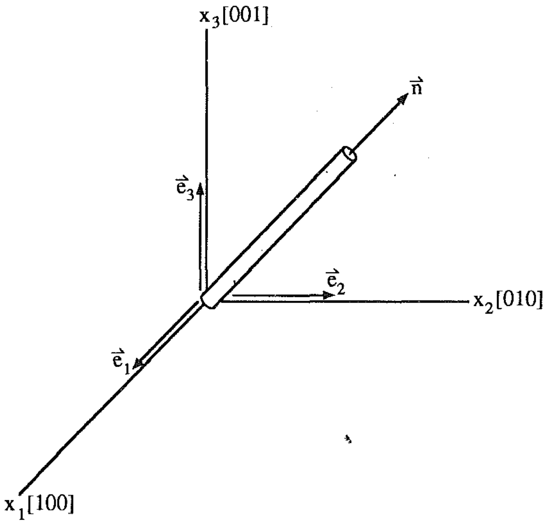

Fig. 2 Filamentary conductor arbitrarily oriented with respect to the principal cystrallographic axes

system (x_1, x_2, x_3) is aligned with the principal symmetry axes of the cubic crystal.

The conduction equations for a stressed cubic crystal with diamond structure are more complex in an off-axis coordinate system (x'_1, x'_2, x'_3) rotated from the principal symmetry axes (x_1, x_2, x_3) as shown in Fig. 1. Since Eq. (49) is form invariant upon coordinate system transformation, the conduction equations in the off-axis coordinate system for a general anisotropic conductor are

$$\begin{aligned} E'_1 &= [\rho_1^{0\,\prime} + \bar{\rho}\pi'_{1\alpha}\sigma'_\alpha]J'_1 + [\rho_6^{0\,\prime} + \bar{\rho}\pi'_{6\alpha}\sigma'_\alpha]J'_2 + [\rho_4^{0\,\prime} + \bar{\rho}\pi'_{4\alpha}\sigma'_\alpha]J'_3 \\ E'_2 &= [\rho_6^{0\,\prime} + \bar{\rho}\pi'_{6\alpha}\sigma'_\alpha]J'_1 + [\rho_2^{0\,\prime} + \bar{\rho}\pi'_{2\alpha}\sigma'_\alpha]J'_2 + [\rho_5^{0\,\prime} + \bar{\rho}\pi'_{5\alpha}\sigma'_\alpha]J'_3 \\ E'_3 &= [\rho_4^{0\,\prime} + \bar{\rho}\pi'_{4\alpha}\sigma'_\alpha]J'_1 + [\rho_5^{0\,\prime} + \bar{\rho}\pi'_{5\alpha}\sigma'_\alpha]J'_2 + [\rho_3^{0\,\prime} + \bar{\rho}\pi'_{3\alpha}\sigma'_\alpha]J'_3 \end{aligned} \tag{51}$$

Upon substitution of the unstressed resistivity components in Eq. (45) into the transformation relations in Eq. (33), it is observed that the unstressed resistivity tensor of a cubic crystal with diamond structure is isotropic (invariant under coordinate system transformation)

$$\begin{aligned} \rho_1^{0\,\prime} &= \rho_2^{0\,\prime} = \rho_3^{0\,\prime} = \bar{\rho} \\ \rho_4^{0\,\prime} &= \rho_5^{0\,\prime} = \rho_6^{0\,\prime} = 0 \end{aligned} \tag{52}$$

Substitution of these expressions into the formulas in Eq. (51) yields the basic off-axis governing equations of conduction for a stressed cubic crystal with diamond structure

$$\begin{aligned} \frac{E'_1}{\bar{\rho}} &= [1 + \pi'_{1\alpha}\sigma'_\alpha]J'_1 + [\pi'_{6\alpha}\sigma'_\alpha]J'_2 + [\pi'_{4\alpha}\sigma'_\alpha]J'_3 \\ \frac{E'_2}{\bar{\rho}} &= [\pi'_{6\alpha}\sigma'_\alpha]J'_1 + [1 + \pi'_{2\alpha}\sigma'_\alpha]J'_2 + [\pi'_{5\alpha}\sigma'_\alpha]J'_3 \\ \frac{E'_3}{\bar{\rho}} &= [\pi'_{4\alpha}\sigma'_\alpha]J'_1 + [\pi'_{5\alpha}\sigma'_\alpha]J'_2 + [1 + \pi'_{3\alpha}\sigma'_\alpha]J'_3 \end{aligned} \tag{53}$$

The primed piezoresistive coefficients in Eq. (53) are to be evaluated for the chosen primed coordinate system by substituting the unprimed values in Eq. (46) into the transformation relations given in Eq. (37). The expressions in Eq. (53) were first presented in the literature by Pfann and Thurston [1961].

Stress-Induced Resistance Changes in One-Dimensional Filamentary Conductors

Introduction. Early applications of semiconductor strain (stress) gages which utilized the piezoresistive effect exhibited by silicon were made by Mason and Thurston [1957], Pfann and Thurston [1961], Dean and Douglas [1962], Tufte et al. [1962], and Thurston [1964]. The sensors in these investigations had the shape of thin filaments or thin sheets, and were bonded to the surface of a structural element of interest. The conduction relations given in Eqs. (50, 53) were utilized to derive relations between the resistance change experienced by the piezoresistive sensor and the state of stress to which the sensor was subjected. The most complicated stress states considered in these prior studies were typically plane stress states because it was presumed that the sensor would be attached to a free (unloaded) surface of the body of interest.

As discussed previously, piezoresistive sensors have recently been applied in experimental structural and reliability analyses of electronic packages. In this case, the sensors are diffused or implanted filamentary resistors fabricated into the surface of a silicon wafer. The structural element of interest in the die itself. Since the sensors are an integral part of the structure, no bonding is necessary. Such sensing chips are often referred to within the industry as stress chips, test chips, and test structures. It is important to note that the die surface in modern electronic packages may not be a free surface, especially if the chip is encapsulated. Therefore, the simplifying assumption of plane stress made in previous mounted gage investigations will be detrimental when establishing formulas appropriate for test chip stress sensors. In the presentation below, new general expressions are obtained for the stress-induced resistance changes which occur in arbitrarily oriented one-dimensional filamentary conductors fabricated in crystals with cubic symmetry and diamond structure.

Coordinate System Along Principal Symmetry Axes. A uniform one-dimensional filamentary conductor at a general orientation with respect to the principal crystallographic directions $x_1 = [100]$, $x_2 = [010]$, and $x_3 = [001]$ of a cubic crystal with diamond structure is shown in Fig. 2. The orientation of the conductor is defined by the unit vector

$$\mathbf{n} = l\mathbf{e}_1 + m\mathbf{e}_2 + n\mathbf{e}_3 \tag{54}$$

directed along its length. Quantities l, m, n are the direction cosines of the conductor orientation with respect to the x_1, x_2, x_3 axes, respectively. When a current is present in the conductor, the net flow of charge is along the length of the conductor so that

$$\mathbf{J} = J\mathbf{n} \text{ or } J_1 = lJ,\ J_2 = mJ,\ J_3 = nJ \tag{55}$$

and

$$I = JA \tag{56}$$

where J is the magnitude of the current density, I is the current, and A is the cross-sectional area of the unstressed conductor (dimensional changes are neglected). Substitution of Eqs. (55, 56) into Eq. (50) yields the governing equations of conduction for the stressed filament

$$\begin{aligned} \frac{E_1 A}{\bar{\rho} I} &= [1 + \pi_{11}\sigma_{11} + \pi_{12}(\sigma_{22} + \sigma_{33})]l + \pi_{44}\sigma_{12}m + \pi_{44}\sigma_{13}n \\ \frac{E_2 A}{\bar{\rho} I} &= [1 + \pi_{11}\sigma_{22} + \pi_{12}(\sigma_{11} + \sigma_{33})]m + \pi_{44}\sigma_{12}l + \pi_{44}\sigma_{23}n \\ \frac{E_3 A}{\bar{\rho} I} &= [1 + \pi_{11}\sigma_{33} + \pi_{12}(\sigma_{11} + \sigma_{22})]n + \pi_{44}\sigma_{13}l + \pi_{44}\sigma_{23}m \end{aligned} \tag{57}$$

The potential drop along the conductor is

$$V = (E_1 l + E_2 m + E_3 n)L \tag{58}$$

where L is the length of the unstressed filament (dimensional changes are again neglected). Substitution of the relations in Eq. (57) into Eq.(58) gives the desired expression for the resistance of the stressed filamentary conductor

$$R^\sigma = \frac{V}{I} = \frac{\bar{\rho}L}{A}[1 + (\pi_{11}\sigma_{11} + \pi_{12}[\sigma_{22} + \sigma_{33}])l^2$$

$$(\pi_{11}\sigma_{22} + \pi_{12}[\sigma_{11} + \sigma_{33}])m^2$$

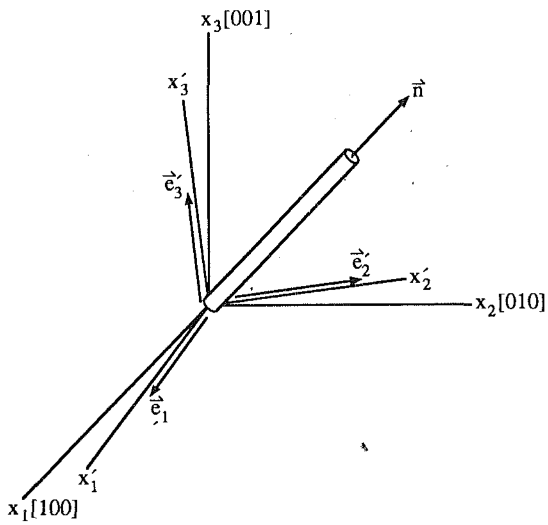

Fig. 3 Filamentary conductor arbitrarily oriented with respect to an off-axis coordinate system

$$+(\pi_{11}\sigma_{33}+\pi_{12}[\sigma_{11}+\sigma_{22}])n^2$$
$$+2\pi_{44}(\sigma_{12}lm+\sigma_{13}ln+\sigma_{23}mn)] \quad (59)$$

The familiar formula for the unstressed resistance of the filamentary conductor is easily obtained by setting the stresses equal to zero in Eq. (59)

$$R=\frac{\bar{\rho}L}{A} \quad (60)$$

Combining these two results yields the normalized change in resistance for an arbitrarily oriented filamentary conductor under stress

$$\frac{\Delta R}{R}=[\pi_{11}\sigma_{22}+\pi_{12}(\sigma_{22}+\sigma_{33})]l^2+[\pi_{11}\sigma_{22}+\pi_{12}(\sigma_{11}+\sigma_{33})]m^2$$
$$+[\pi_{11}\sigma_{33}+\pi_{12}(\sigma_{11}+\sigma_{22})]n^2+2\pi_{44}[\sigma_{12}lm+\sigma_{13}ln+\sigma_{23}mn] \quad (61)$$

Off-Axis Coordinate System. It is often convenient to have the resistance change expressed in terms of stress components which are resolved in a coordinate system other than one aligned with the principal crystallographic directions. A uniform one-dimensional filamentary conductor at a general orientation with respect to primed coordinate system (x_1', x_2', x_3') is shown in Fig. 3. The primed system is itself rotated with respect to the unprimed coordinate system $x_1=[100]$, $x_2=[010]$, and $x_3=[001]$ along the symmetry axes of a cubic crystal with diamond structure. As before, the orientation of the conductor is defined by a unit vector

$$\mathbf{n}=l'\mathbf{e}_1'+m'\mathbf{e}_2'+n'\mathbf{e}_3' \quad (62)$$

directed along its length. Quantities l', m', n' are the direction cosines of the conductor orientation with respect to the x_1', x_2', x_3' axes, respectively.

As in the prior derivation, the net charge flow in the filament is along its length. Thus, the current density vector is proportional to the normal vector. With this observation, the off-axis governing equations of conduction for a stressed cubic crystal with diamond structure given in Eq. (53) simplify to

$$\frac{E_1'A}{\bar{\rho}I}=[1+\pi_{1\alpha}'\sigma_\alpha']l'+[\pi_{6\alpha}'\sigma_\alpha']m'+[\pi_{4\alpha}'\sigma_\alpha']n'$$
$$\frac{E_2'A}{\bar{\rho}I}=[\pi_{6\alpha}'\sigma_\alpha']l'+[1+\pi_{2\alpha}'\sigma_\alpha']m'+[\pi_{5\alpha}'\sigma_\alpha']n'$$
$$\frac{E_3'A}{\bar{\rho}I}=[\pi_{4\alpha}'\sigma_\alpha']l'+[\pi_{5\alpha}'\sigma_\alpha']m'+[1+\pi_{3\alpha}'\sigma_\alpha']n' \quad (63)$$

As before, dimensional changes have been neglected.

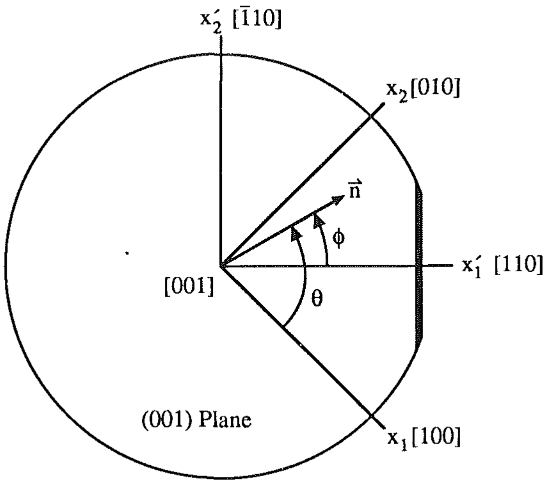

Fig. 4 (100) Silicon wafer geometry

In terms of the electric field vector components in the primed coordinate system, the potential drop along the conductor is

$$V=(E_1'l'+E_2'm'+E_3'n')L \quad (64)$$

where L is the length of the unstressed filament and dimensional changes are again neglected. Substitution of the expressions in Eq. (63) into Eq. (64) gives the formula for the resistance of the stressed filamentary conductor in terms of the stress components in the primed system

$$R^\sigma=\frac{V}{I}=\frac{\bar{\rho}L}{A}[1+(\pi_{1\alpha}'\sigma_\alpha')l'^2+(\pi_{2\alpha}'\sigma_\alpha')m'^2$$
$$+(\pi_{3\alpha}'\sigma_\alpha')n'^2+2(\pi_{4\alpha}'\sigma_\alpha')l'n'$$
$$+2(\pi_{5\alpha}'\sigma_\alpha')m'n'+2(\pi_{6\alpha}'\sigma_\alpha')l'm'] \quad (65)$$

Setting the stress components equal to zero in this equation leads to the same unstressed resistance value given in Eq. (60). Using Eqs. (60, 65), the normalized change in resistance for an arbitrarily oriented filamentary conductor in terms of the off-axis stress components is

$$\frac{\Delta R}{R}=(\pi_{1\alpha}'\sigma_\alpha')l'^2+(\pi_{2\alpha}'\sigma_\alpha')m'^2+(\pi_{3\alpha}'\sigma_\alpha')n'^2$$
$$+2(\pi_{4\alpha}'\sigma_\alpha')l'n'+2(\pi_{5\alpha}'\sigma_\alpha')m'n'+2(\pi_{6\alpha}'\sigma_\alpha')l'm' \quad (66)$$

The primed piezoresistive coefficients in Eq. (66) are evaluated by substituting the unprimed values in Eq. (46) into the transformation relations given in Eq. (37). This calculation is routine once the precise orientation of the primed coordinate system have been specified. The expressions given in Eqs. (61, 66) are the general relations needed for design and application of piezoresistive stress sensors on semiconductor test chips.

Applications Using (100) Silicon Wafers

General Equations for Sensors on (100) Wafers. A general (100) oriented silicon wafer is shown in Fig. 4. The surface of the wafer has been taken to be the (001) plane. Therefore, the $x_1=[100]$ and $x_2=[010]$ axes lie in the plane of the wafer, and the $x_3=[001]$ axis is normal to the wafer. The general expression for the resistance change of a filamentary piezoresistive sensor in the plane of the wafer can be obtained by setting $n=0$ in Eq. (61)

$$\frac{\Delta R}{R}=[\pi_{11}\sigma_{11}+\pi_{12}(\sigma_{22}+\sigma_{33})]l^2$$
$$+[\pi_{11}\sigma_{22}+\pi_{12}(\sigma_{11}+\sigma_{33})]m^2+2\pi_{44}[\sigma_{12}lm] \quad (67)$$

where

$$l=\cos\theta \qquad m=\sin\theta \quad (68)$$

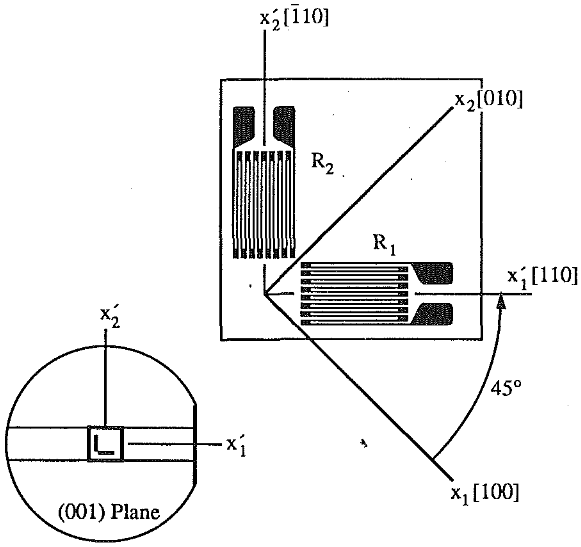

Fig. 5 Two element 0-90 rosette

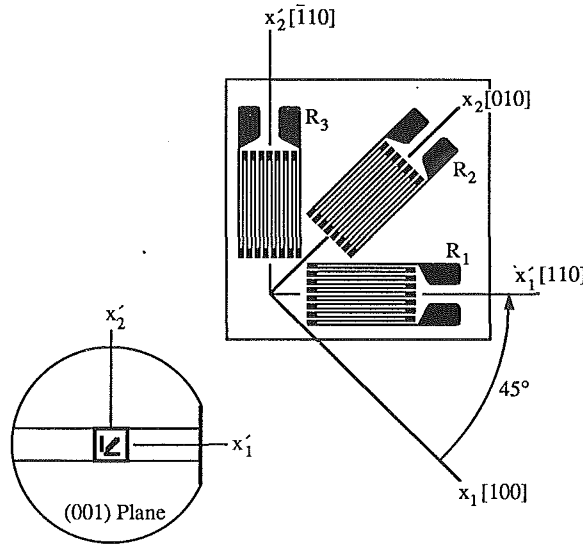

Fig. 6 Three element 0-45-90 rosette

and θ is the angle between the x_1-axis and the resistor orientation (see Fig. 4). Equation (67) indicates that the out-of-plane shear stresses σ_{13} and σ_{23} do not influence the resistances of stress sensors fabricated on (100) silicon wafers. This means that a sensor rosette on (100) silicon can at best measure four of the six unique components of the stress tensor.

As shown in Fig. 4, the axes $x_1=[100]$ and $x_2=[010]$ are rotated by 45 deg from a more natural wafer coordinate system x'_1, x'_2 where the axes are parallel and perpendicular to the primary wafer flat. The resistance change of an in-plane sensor can also be expressed in terms of stress components resolved in the primed coordinate system using Eq. (66). The off-axis piezoresistive coefficients in the primed coordinate system must be first evaluated by substituting the unprimed values in Eq. (46) and the appropriate direction cosines into the transformation relations given in Eq. (37). For the unprimed and primed coordinate systems shown in Fig. 4, the direction cosines are

$$[a_{ij}]=\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{69}$$

Substitution of the off-axis piezoresistive coefficients calculated in the manner described above into Eq. (66) yields

$$\begin{aligned}\frac{\Delta R}{R}=&\left[\left(\frac{\pi_{11}+\pi_{12}+\pi_{44}}{2}\right)\sigma'_{11}+\left(\frac{\pi_{11}+\pi_{12}-\pi_{44}}{2}\right)\sigma'_{22}\right]l'^2\\ &+\left[\left(\frac{\pi_{11}+\pi_{12}-\pi_{44}}{2}\right)\sigma'_{11}+\left(\frac{\pi_{11}+\pi_{12}+\pi_{44}}{2}\right)\sigma'_{22}\right]m'^2\\ &+\pi_{12}\sigma'_{33}+2(\pi_{11}-\pi_{12})\sigma'_{12}l'm'\end{aligned} \tag{70}$$

where

$$l'=\cos\phi \qquad m'=\sin\phi \tag{71}$$

and ϕ is the angle between the x'_1-axis and the resistor orientation.

Two-Element 0-90 Rosette. A two element sensor rosette fabricated on (100) silicon with the resistors at angles of 0 and 90 deg from the x'_1-axis is shown in Fig. 5. As is typical with commercially available strain gages, a serpentine pattern has been used for the sensing elements. This two resistor configuration has been utilized previously by several investigators including Spencer et al. [1981], Edwards and co-workers [1983, 1987], Beaty et al. [1990]. By applying Eq. (70) to each of the two resistor orientations, the stress-induced resistance changes are found to be

$$\begin{aligned}\frac{\Delta R_1}{R_1}&=\left(\frac{\pi_{11}+\pi_{12}+\pi_{44}}{2}\right)\sigma'_{11}+\left(\frac{\pi_{11}+\pi_{12}-\pi_{44}}{2}\right)\sigma'_{22}+\pi_{12}\sigma'_{33}\\ \frac{\Delta R_2}{R_2}&=\left(\frac{\pi_{11}+\pi_{12}-\pi_{44}}{2}\right)\sigma'_{11}+\left(\frac{\pi_{11}+\pi_{12}+\pi_{44}}{2}\right)\sigma'_{22}+\pi_{12}\sigma'_{33}\end{aligned} \tag{72}$$

For plane stress situations ($\sigma'_{33}=0$), these equations can be inverted to solve for the in-plane normal stress components in terms of the measured resistance changes

$$\begin{aligned}\sigma'_{11}&=\frac{\pi_{44}\left[\frac{\Delta R_1}{R_1}+\frac{\Delta R_2}{R_2}\right]+(\pi_{11}+\pi_{12})\left[\frac{\Delta R_1}{R_1}-\frac{\Delta R_2}{R_2}\right]}{2\pi_{44}(\pi_{11}+\pi_{12})}\\ \sigma'_{22}&=\frac{\pi_{44}\left[\frac{\Delta R_1}{R_1}+\frac{\Delta R_2}{R_2}\right]-(\pi_{11}+\pi_{12})\left[\frac{\Delta R_1}{R_1}-\frac{\Delta R_2}{R_2}\right]}{2\pi_{44}(\pi_{11}+\pi_{12})}\end{aligned} \tag{73}$$

The expressions in Eq. (73) indicate that successful utilization of this sensor requires accurate values of π_{44} and the combined piezoresistive parameter $(\pi_{11}+\pi_{12})$. These constants can be measured using a uniaxial loading calibration procedure if the wafer is fabricated into specimen strips as indicated in Fig. 5. When a known uniaxial stress $\sigma'_{11}=\sigma$ is applied to the strip in the x'_1-direction, the relations in Eq. (72) reduce to

$$\begin{aligned}\frac{\Delta R_1}{R_1}&=\left(\frac{\pi_{11}+\pi_{12}+\pi_{44}}{2}\right)\sigma\\ \frac{\Delta R_2}{R_2}&=\left(\frac{\pi_{11}+\pi_{12}-\pi_{44}}{2}\right)\sigma\end{aligned} \tag{74}$$

Therefore, the linearly independent combined piezoresistive parameters $(\pi_{11}+\pi_{12}+\pi_{44})$ and $(\pi_{11}+\pi_{12}-\pi_{44})$ can be evaluated experimentally through controlled application of a uniaxial loading. The values of π_{44} and $(\pi_{11}+\pi_{12})$ can then be found by algebraic manipulation of these combined parameters. Experimental application of uniaxial stress states to wafer strips can be performed using a four point bending loading fixture as demonstrated by Gee et al. [1988], and Beaty and co-workers [1990].

Three Element 0-45-90 Rosette. The two element rosette presented in the last section can be improved upon if a third sensor is included. Such a three element rosette with the additional resistor at 45 deg from the x'_1-axis is shown in Fig. 6. A sensor rosette incorporating this three element configuration and a fourth resistor at 135 deg from the x'_1-axis was previously applied using (100) silicon by Natarajan and Bhattacharyya [1986]. As will be shown below, no additional stress components can be measured by including the fourth resistor. Therefore, the three element design should suffice for most applications.

Repeated application of Eq. (70) to each of the piezoresistive sensing element leads to the following expressions for the stress-induced resistance changes:

$$\frac{\Delta R_1}{R_1}=\left(\frac{\pi_{11}+\pi_{12}+\pi_{44}}{2}\right)\sigma'_{11}+\left(\frac{\pi_{11}+\pi_{12}-\pi_{44}}{2}\right)\sigma'_{22}+\pi_{12}\sigma'_{33}$$

$$\frac{\Delta R_2}{R_2}=\left(\frac{\pi_{11}+\pi_{12}}{2}\right)(\sigma'_{11}+\sigma'_{22})+\pi_{12}\sigma'_{33}+(\pi_{11}-\pi_{12})\sigma'_{12}$$

$$\frac{\Delta R_3}{R_3}=\left(\frac{\pi_{11}+\pi_{12}-\pi_{44}}{2}\right)\sigma'_{11}+\left(\frac{\pi_{11}+\pi_{12}+\pi_{44}}{2}\right)\sigma'_{22}+\pi_{12}\sigma'_{33} \quad (75)$$

For plane stress situations ($\sigma'_{33}=0$), these equations can be inverted to solve for the three in-plane stress components in terms of the measured resistance changes

$$\sigma'_{11}=\frac{\pi_{44}\left[\frac{\Delta R_1}{R_1}+\frac{\Delta R_3}{R_3}\right]+(\pi_{11}+\pi_{12})\left[\frac{\Delta R_1}{R_1}-\frac{\Delta R_3}{R_3}\right]}{2\pi_{44}(\pi_{11}+\pi_{12})}$$

$$\sigma'_{22}=\frac{\pi_{44}\left[\frac{\Delta R_1}{R_1}+\frac{\Delta R_3}{R_3}\right]-(\pi_{11}+\pi_{12})\left[\frac{\Delta R_1}{R_1}-\frac{\Delta R_3}{R_3}\right]}{2\pi_{44}(\pi_{11}+\pi_{12})}$$

$$\sigma'_{12}=\left(\frac{1}{\pi_{11}-\pi_{12}}\right)\left[\frac{\Delta R_2}{R_2}-\frac{1}{2}\left(\frac{\Delta R_1}{R_1}+\frac{\Delta R_3}{R_3}\right)\right] \quad (76)$$

Values for all three of the piezoresistive coefficients must be found to utilize the expressions in Eq. (76). These constants can be measured using uniaxial loading and hydrostatic pressure calibration procedures. If the wafer is cut into specimen strips as indicated in Fig. 6 and a known uniaxial stress $\sigma'_{11}=\sigma$ is applied in the x'_1-direction, the expressions in Eq. (75) reduce to

$$\frac{\Delta R_1}{R_1}=\left(\frac{\pi_{11}+\pi_{12}+\pi_{44}}{2}\right)\sigma$$

$$\frac{\Delta R_2}{R_2}=\left(\frac{\pi_{11}+\pi_{12}}{2}\right)\sigma$$

$$\frac{\Delta R_3}{R_3}=\left(\frac{\pi_{11}+\pi_{12}-\pi_{44}}{2}\right)\sigma \quad (77)$$

With a controlled application of uniaxial stress, the relations in Eq. (77) can be manipulated to determine the values of π_{44} and $(\pi_{11}+\pi_{12})$. Application of an additional state of stress is required to complete the calibration procedure. If this stress state is chosen to be a hydrostatic pressure ($\sigma'_{11}=\sigma'_{22}=\sigma'_{33}=-p$), the expressions in Eq. (75) become

$$\frac{\Delta R_1}{R_1}=\frac{\Delta R_2}{R_2}=\frac{\Delta R_3}{R_3}=-[\pi_{11}+2\pi_{12}]p \quad (78)$$

With a controlled application of a hydrostatic loading, Eq. (78) can be used to determine the value of $(\pi_{11}+2\pi_{12})$. The individual values of π_{11}, π_{12}, π_{44} can then be obtained by algebraic manipulation of the combined piezoresistive parameters obtained with the uniaxial loading and hydrostatic pressure calibration procedures.

Off-Axis Three Element 0-45-90 Rosette. The three element rosette presented in the last section could not be calibrated with a single uniaxial loading test when the wafer was cut into strips as indicated in Fig. 6. Detailed analysis of the expressions in Eq. (75) has indicated that a single uniaxial tension calibration procedure cannot yield all three of the needed piezoresistive coefficients when the wafer is cut into strips along any of the unprimed or primed coordinate directions shown in Fig. 6. Only uniaxial testing at an odd orientation (e.g. $\theta=53$ deg) will provide an easy one step calibration of this sensor rosette. Such a procedure will require slicing the wafer into strips at the odd orientation. Therefore, the three element sensor rosette will not line up conveniently with the edges of the rectangular test chips eventually cut from these odd angle wafer strips.

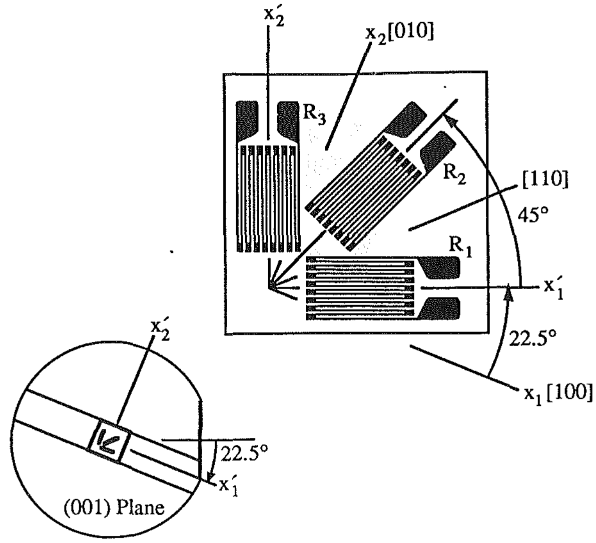

Fig. 7 Off-axis three element 0-45-90 rosette

Although not a major inconvenience, the difficulties discussed above can be avoided if an off-axis three element sensor rosette is utilized, and the wafer strips are cut parallel and perpendicular to the resistors in the rosette. Such a sensor rosette is shown in Fig. 7, where the x'_1, x'_2 axes are rotated by 22.5 deg from the principal crystallographic directions. Repeated application of Eq. (66) to the three resistors in Fig. 7 yields an algebraically complicated set of linear equations for the resistance changes in terms of the stress components evaluated in the off-axis primed coordinate system. For the case of plane stress ($\sigma'_{33}=0$), inversion of this linear system leads to desired expressions for the stress components

$$\sigma'_{11}=\left[\frac{\pi_{11}}{\pi_{11}^2-\pi_{12}^2}\right]\frac{\Delta R_1}{R_1}+\frac{1}{2}\left[\frac{\pi_{11}-\pi_{12}-\pi_{44}}{\pi_{44}(\pi_{11}-\pi_{12})}\right]\frac{\Delta R_2}{R_2}+\frac{1}{2}\left[\frac{\pi_{44}-\pi_{11}-\pi_{12}}{\pi_{44}(\pi_{11}+\pi_{12})}\right]\frac{\Delta R_3}{R_3}$$

$$\sigma'_{22}=-\left[\frac{\pi_{11}}{\pi_{11}^2-\pi_{12}^2}\right]\frac{\Delta R_1}{R_1}-\frac{1}{2}\left[\frac{\pi_{11}-\pi_{12}-\pi_{44}}{\pi_{44}(\pi_{11}-\pi_{12})}\right]\frac{\Delta R_2}{R_2}+\frac{1}{2}\left[\frac{\pi_{44}+\pi_{11}+\pi_{12}}{\pi_{44}(\pi_{11}+\pi_{12})}\right]\frac{\Delta R_3}{R_3}$$

$$\sigma'_{12}=-\frac{1}{2}\left[\frac{1}{\pi_{11}^2-\pi_{12}^2}\right]\frac{\Delta R_1}{R_1}+\frac{1}{2}\left[\frac{\pi_{11}-\pi_{12}+\pi_{44}}{\pi_{44}(\pi_{11}-\pi_{12})}\right]\frac{\Delta R_2}{R_2}-\frac{1}{2\pi_{44}}\frac{\Delta R_3}{R_3} \quad (79)$$

Accurate values for all three of the piezoresistive coefficients must be found before the relations in Eq. (79) can be applied. These constants can be measured using a single uniaxial loading calibration step if the wafer is cut into specimen strips as indicated in Fig. 7, and a known uniaxial stress $\sigma'_{11}=\sigma$ is applied in the x'_1-direction. Using Eq. (66), it can be shown that the resistance changes for this loading are

$$\frac{\Delta R_1}{R_1}=\left(\frac{3\pi_{11}+\pi_{12}+\pi_{44}}{4}\right)\sigma$$

$$\frac{\Delta R_2}{R_2}=\left(\frac{\pi_{11}+3\pi_{12}+\pi_{44}}{4}\right)\sigma$$

$$\frac{\Delta R_3}{R_3}=\left(\frac{\pi_{11}+3\pi_{12}-\pi_{44}}{4}\right)\sigma \qquad (80)$$

With a controlled application of uniaxial stress, the relations in Eq. (80) can be used to determine three linearly independent combined piezoresistive parameters. These parameters can then be algebraically manipulated to find the individual values of π_{11}, π_{12}, π_{44}.

Rosettes Capable of Measuring Four Stress Components. From Eq. (67), it is seen that the resistance change of an in-plane sensor fabricated on (100) silicon depends on four components of stress (σ_{11}, σ_{22}, σ_{33}, σ_{12}). The rosettes discussed above have not been able to measure the out-of-plane normal stress σ_{33}. It seems natural to assume that the potential exists to design a four element rosette capable of measuring four stress components. Repeated application of Eq. (67) to four arbitrarily oriented piezoresistive sensors yields the following equations for the resistance changes:

$$\begin{bmatrix}\frac{\Delta R_1}{R_1}\\ \frac{\Delta R_2}{R_2}\\ \frac{\Delta R_3}{R_3}\\ \frac{\Delta R_4}{R_4}\end{bmatrix}=\begin{bmatrix} l_1^2 & m_1^2 & 1 & l_1m_1\\ l_2^2 & m_2^2 & 1 & l_2m_2\\ l_3^2 & m_3^2 & 1 & l_3m_3\\ l_4^2 & m_4^2 & 1 & l_4m_4\end{bmatrix}\begin{bmatrix}\pi_{11}\sigma_{11}+\pi_{12}\sigma_{22}\\ \pi_{12}\sigma_{11}+\pi_{11}\sigma_{22}\\ \pi_{12}\sigma_{33}\\ 2\pi_{44}\sigma_{12}\end{bmatrix} \qquad (81)$$

where l_i, m_i are the direction cosines for the orientation of the ith resistor ($i=1, 2, 3, 4$). It is easily demonstrated that the determinant of the coefficient matrix in Eq. (81) is zero for all possible choices of the direction cosines. Therefore, it is not possible to invert the system of equations, and no four element rosette on (100) silicon can be found which is capable of measuring four stress components.

The above discussion pertains to rosettes formed with identically doped sensing resistors. A four element sensor rosette which contains two n-type resistors and two p-type resistors has been designed and applied by Miura et al. [1987, 1990]. The configuration of the sensing elements is illustrated in Fig. 8. This sensor is capable of measuring four stress components because the piezoresistive coefficients of the n-type and p-type resistor are different.

Repeated application of Eq. (70) to each of the rosette elements in Fig. 8 leads to the following expressions for the stress-induced resistance changes:

$$\frac{\Delta R_1}{R_1}=\left(\frac{\pi_{11}^n+\pi_{12}^n+\pi_{44}^n}{2}\right)\sigma'_{11}+\left(\frac{\pi_{11}^n+\pi_{12}^n-\pi_{44}^n}{2}\right)\sigma'_{22}+\pi_{12}^n\sigma'_{33}$$

$$\frac{\Delta R_2}{R_2}=\left(\frac{\pi_{11}^n+\pi_{12}^n-\pi_{44}^n}{2}\right)\sigma'_{11}+\left(\frac{\pi_{11}^n+\pi_{12}^n+\pi_{44}^n}{2}\right)\sigma'_{22}+\pi_{12}^n\sigma'_{33}$$

$$\frac{\Delta R_3}{R_3}=\left(\frac{\pi_{11}^p+\pi_{12}^p}{2}\right)(\sigma'_{11}+\sigma'_{22})+\pi_{12}^p\sigma'_{33}-(\pi_{11}^p-\pi_{12}^p)\sigma'_{12}$$

$$\frac{\Delta R_4}{R_4}=\left(\frac{\pi_{11}^p+\pi_{12}^p}{2}\right)(\sigma'_{11}+\sigma'_{22})+\pi_{12}^p\sigma'_{33}+(\pi_{11}^p-\pi_{12}^p)\sigma'_{12} \qquad (82)$$

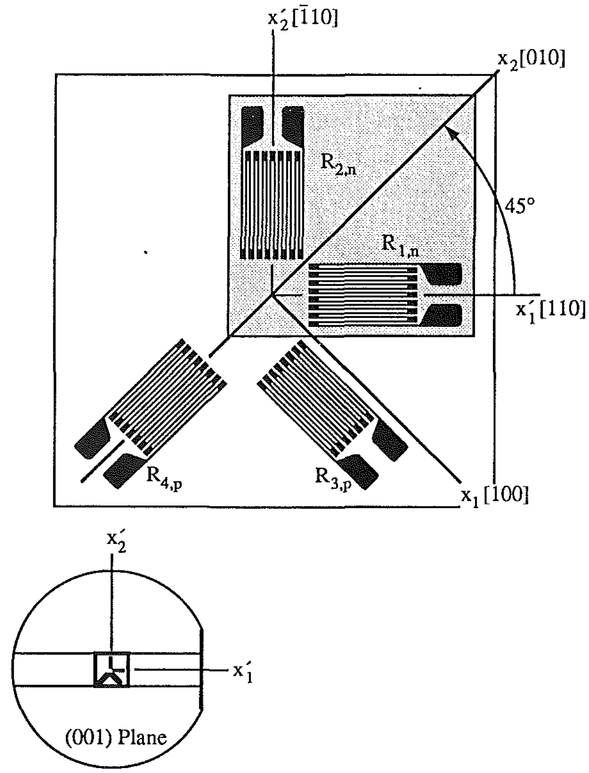

Fig. 8 Four element sensor with p-type and n-type resistors

The n and p superscripts on the piezoresistive coefficients in Eq. (82) denote n-type and p-type resistors, respectively. These equations can be inverted to solve for the three in-plane stress components and the out-of-plane normal stress component. This calculation gives

$$\sigma'_{11}=\frac{\pi_{12}^n\left[\frac{\Delta R_3}{R_3}+\frac{\Delta R_4}{R_4}\right]-\pi_{12}^p\left[\frac{\Delta R_1}{R_1}+\frac{\Delta R_2}{R_2}\right]}{2(\pi_{11}^p\pi_{12}^n-\pi_{11}^n\pi_{12}^p)}+\frac{1}{2\pi_{44}^n}\left[\frac{\Delta R_1}{R_1}-\frac{\Delta R_2}{R_2}\right]$$

$$\sigma'_{22}=\frac{\pi_{12}^n\left[\frac{\Delta R_3}{R_3}+\frac{\Delta R_4}{R_4}\right]-\pi_{12}^p\left[\frac{\Delta R_1}{R_1}+\frac{\Delta R_2}{R_2}\right]}{2(\pi_{11}^p\pi_{12}^n-\pi_{11}^n\pi_{12}^p)}-\frac{1}{2\pi_{44}^n}\left[\frac{\Delta R_1}{R_1}-\frac{\Delta R_2}{R_2}\right]$$

$$\sigma'_{33}=\frac{(\pi_{11}^p+\pi_{12}^p)\left[\frac{\Delta R_1}{R_1}+\frac{\Delta R_2}{R_2}\right]-(\pi_{11}^n+\pi_{12}^n)\left[\frac{\Delta R_3}{R_3}+\frac{\Delta R_4}{R_4}\right]}{2(\pi_{11}^p\pi_{12}^n-\pi_{11}^n\pi_{12}^p)}$$

$$\sigma'_{12}=-\frac{1}{2(\pi_{11}^p-\pi_{12}^p)}\left[\frac{\Delta R_3}{R_3}-\frac{\Delta R_4}{R_4}\right] \qquad (83)$$

The expressions in Eq. (83) require accurate values for five piezoresistive coefficients (π_{11}^n, π_{12}^n, π_{44}^n, π_{11}^p, π_{12}^p). These constants can be obtained using uniaxial loading and hydrostatic pressure calibration procedures. If the wafer is cut into specimen strips as indicated in Fig. 8 and a known uniaxial stress $\sigma'_{11}=\sigma$ is applied to the x'_1-direction, the expressions of Eq. (82) reduce to

$$\frac{\Delta R_1}{R_1}=\left(\frac{\pi_{11}^n+\pi_{12}^n+\pi_{44}^n}{2}\right)\sigma$$

$$\frac{\Delta R_2}{R_2}=\left(\frac{\pi_{11}^n+\pi_{12}^n-\pi_{44}^n}{2}\right)\sigma$$

$$\frac{\Delta R_3}{R_3}=\frac{\Delta R_4}{R_4}=\left(\frac{\pi_{11}^p+\pi_{12}^p}{2}\right)\sigma \qquad (84)$$

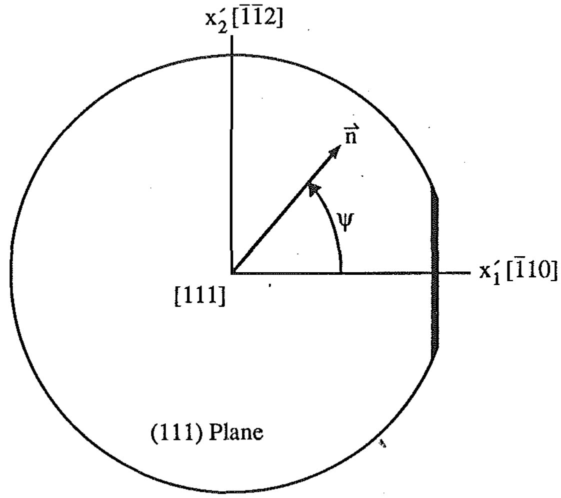

Fig. 9 (111) Silicon wafer geometry

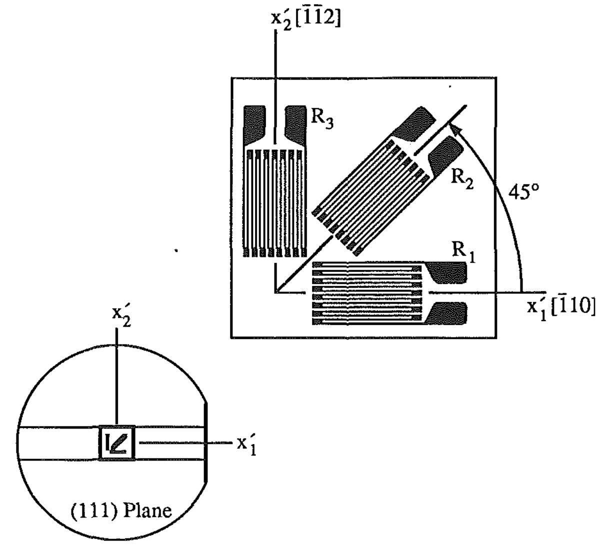

Fig. 10 Three element 0-45-90 rosette for (111) silicon

With a controlled application of uniaxial stress, the relations in Eq. (84) can be manipulated to determine the values of π^n_{44}, $(\pi^n_{11}+\pi^n_{12})$, and $(\pi^p_{11}+\pi^p_{12})$. Application of an additional state of stress is required to complete the calibration procedure. If this stress state is chosen to be a hydrostatic pressure $(\sigma'_{11}=\sigma'_{22}=\sigma'_{33}=-p)$, the expressions in Eq. (82) become

$$\frac{\Delta R_1}{R_1}=\frac{\Delta R_2}{R_2}=-[\pi^n_{11}+2\pi^n_{12}]p$$

$$\frac{\Delta R_3}{R_3}=\frac{\Delta R_4}{R_4}=-[\pi^p_{11}+2\pi^p_{12}]p \qquad (85)$$

With a controlled application of a hydrostatic loading, Eq. (85) can be used to determine the values of $(\pi^n_{11}+2\pi^n_{12})$ and $(\pi^p_{11}+2\pi^p_{12})$. The individual values of the five needed piezoresistive coefficients can be obtained by manipulating the combined parameters obtained using the uniaxial loading and hydrostatic pressure calibration procedures.

Applications Using (111) Silicon Wafers

General Equations for Sensors on (111) Wafers. A general (111) silicon wafer is shown in Fig. 9. The surface of the wafer is a (111) plane, and the [111] direction is normal to the wafer plane. The principal crystallographic axes $x_1=[100]$, $x_2=[010]$, and $x_3=[001]$ do not lie in the wafer plane and have not been indicated. For all analyses, a natural wafer coordinate system $x'_1=[\bar{1}, 1, 0]$, $x'_2=[\bar{1}, \bar{1}, 2]$, $x'_3=[1, 1, 1]$ has been adopted. The x'_1, x'_2 axes are parallel and perpendicular to the primary wafer flat.

Using Eq. (66), the resistance change of an arbitrarily oriented in-plane sensor can be expressed in terms of the stress components resolved in the primed coordinate system. The off-axis piezoresistive coefficients in the primed coordinate system must be first evaluated by substituting the unprimed values in Eq. (46) and the appropriate direction cosines into the transformation relations given in Eq. (37). For the described unprimed and primed coordinate systems, the direction cosines are

$$[a_{ij}]=\begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \qquad (86)$$

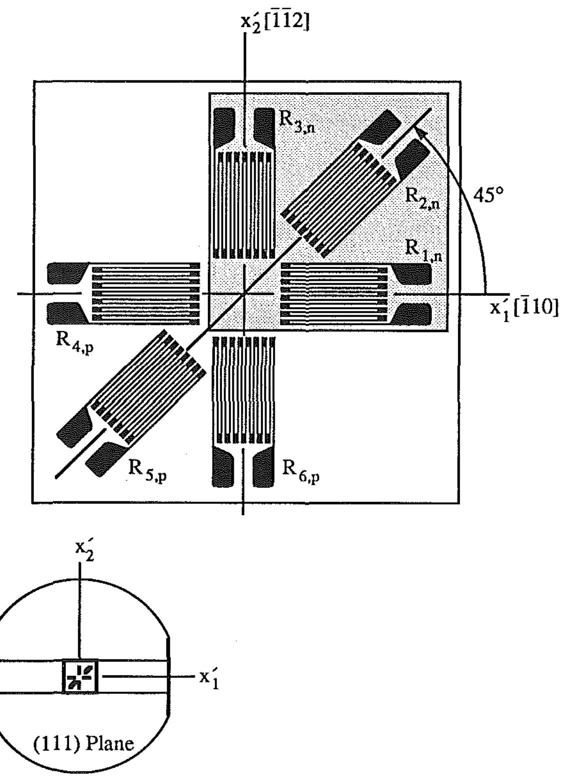

Fig. 11 Six element rosette capable of measuring complete three-dimensional stress states

Substitution of the off-axis piezoresistive coefficients, calculated in the manner described above, into Eq. (66) yields

$$\frac{\Delta R}{R}=[B_1\sigma'_{11}+B_2\sigma'_{22}+B_3\sigma'_{33}+2\sqrt{2}(B_2-B_3)\sigma'_{23}]l'^2$$
$$+[B_2\sigma'_{11}+B_1\sigma'_{22}+B_3\sigma'_{33}-2\sqrt{2}(B_2-B_3)\sigma'_{23}]m'^2$$
$$+[4\sqrt{2}(B_2-B_3)\sigma'_{13}+2(B_1-B_2)\sigma'_{12}]l'm' \qquad (87)$$

where

$$l'=\cos\psi \qquad m'=\sin\psi \qquad (88)$$

are the directions cosines of the resistor orientation with respect to the x'_1, x'_2 axes, and ψ is the angle between the x'_1-axis and the resistor orientation. The coefficients

$$B_1 = \frac{\pi_{11} + \pi_{12} + \pi_{44}}{2}$$

$$B_2 = \frac{\pi_{11} + 5\pi_{12} - \pi_{44}}{6}$$

$$B_3 = \frac{\pi_{11} + 2\pi_{12} - \pi_{44}}{3} \qquad (89)$$

are a set of linearly independent combined piezoresistive parameters. Equation (87) indicates that the resistance change for a resistor in the (111) plane is dependent on all six of the unique stress components. Therefore, the potential exists for developing a sensor rosette which can measure the complete three-dimensional state of stress at points on the surface of a die.

Three Element 0-45-90 Rosette. A three element rosette suitable for evaluating plane stress states on the surface of a die fabricated using (111) silicon is shown in Fig. 10. The sensing elements make angles of 0, 45, and 90 deg from the x'_1-axis. A similar rosette incorporating this three element configuration and a fourth resistor at 135 deg from the x'_1-axis was previously applied using (111) silicon by Gee et al. [1988, 1989].

Repeated application of Eq. (87) to each of the piezoresistive sensing elements leads to the following expressions for the stress-induced resistance changes:

$$\frac{\Delta R_1}{R_1} = B_1\sigma'_{11} + B_2\sigma'_{22} + B_3\sigma'_{33} + 2\sqrt{2}(B_2 - B_3)\sigma'_{23}$$

$$\frac{\Delta R_2}{R_2} = \left(\frac{B_1 + B_2}{2}\right)(\sigma'_{11} + \sigma'_{22}) + B_3\sigma'_{33} + 2\sqrt{2}(B_2 - B_3)\sigma'_{13} + (B_1 - B_2)\sigma'_{12}$$

$$\frac{\Delta R_3}{R_3} = B_2\sigma'_{11} + B_1\sigma'_{22} + B_3\sigma'_{33} - 2\sqrt{2}(B_2 - B_3)\sigma'_{23} \qquad (90)$$

For plane stress situations ($\sigma'_{13} = \sigma'_{23} = \sigma'_{33} = 0$), these equations can be inverted to solve for all three of the in-plane stress components in terms of the measured resistance changes

$$\sigma'_{11} = \left(\frac{1}{B_1 + B_2}\right)\left[\frac{\Delta R_1}{R_1} + \frac{\Delta R_3}{R_3}\right] + \frac{1}{2}\left(\frac{1}{B_1 - B_2}\right)\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_3}{R_3}\right]$$

$$\sigma'_{22} = \left(\frac{1}{B_1 + B_2}\right)\left[\frac{\Delta R_1}{R_1} + \frac{\Delta R_3}{R_3}\right] - \frac{1}{2}\left(\frac{1}{B_1 - B_2}\right)\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_3}{R_3}\right]$$

$$\sigma'_{12} = \frac{1}{2}\left(\frac{1}{B_1 - B_2}\right)\left[\frac{\Delta R_1}{R_1} - 2\frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3}\right] \qquad (91)$$

The expressions in Eq. (91) indicate that both of the combined piezoresistive parameters B_1 and B_2 must be determined prior to using the sensor. These constants can be measured using a uniaxial loading calibration procedure. If the wafer is cut into specimen strips as indicated in Fig. 10 and a known uniaxial stress $\sigma'_{11} = \sigma$ is applied to the x'_1-direction, the expressions in Eq. (90) reduce to

$$\frac{\Delta R_1}{R_1} = B_1\sigma \quad \frac{\Delta R_2}{R_2} = \left(\frac{B_1 + B_2}{2}\right)\sigma \quad \frac{\Delta R_3}{R_3} = B_2\sigma \qquad (92)$$

From these expressions, it is clear that the constants B_1 and B_2 can be easily determined using a controlled application of uniaxial stress.

Complete Stress State Sensor. As mentioned previously, the resistance change of an in-plane sensor fabricated on (111) silicon depends on all six of the unique stress components at a point. A new six element sensor rosette capable of measuring complete three-dimensional states of stress at a point on the surface of a die is shown in Fig. 11. The rosette contains three n-type resistors and three p-type resistors. The sensing elements of each doping type are oriented in directions of $\psi = 0$, 45, and 90 deg from the x'_1-axis.

Repeated application of Eq. (87) to each of the piezoresistive sensing elements leads to the following expressions for the stress-induced resistance changes:

$$\frac{\Delta R_1}{R_1} = B_1^n\sigma'_{11} + B_2^n\sigma'_{22} + B_3^n\sigma'_{33} + 2\sqrt{2}(B_2^n - B_3^n)\sigma'_{23}$$

$$\frac{\Delta R_2}{R_2} = \left(\frac{B_1^n + B_2^n}{2}\right)(\sigma'_{11} + \sigma'_{22}) + B_3^n\sigma'_{33} + 2\sqrt{2}(B_2^n - B_3^n)\sigma'_{13} + (B_1^n - B_2^n)\sigma'_{12}$$

$$\frac{\Delta R_3}{R_3} = B_2^n\sigma'_{11} + B_1^n\sigma'_{22} + B_3^n\sigma'_{33} - 2\sqrt{2}(B_2^n - B_3^n)\sigma'_{23}$$

$$\frac{\Delta R_4}{R_4} = B_1^p\sigma'_{11} + B_2^p\sigma'_{22} + B_3^p\sigma'_{33} + 2\sqrt{2}(B_2^p - B_3^p)\sigma'_{23}$$

$$\frac{\Delta R_5}{R_5} = \left(\frac{B_1^p + B_2^p}{2}\right)(\sigma'_{11} + \sigma'_{22}) + B_3^p\sigma'_{33} + 2\sqrt{2}(B_2^p - B_3^p)\sigma'_{13} + (B_1^p - B_2^p)\sigma'_{12}$$

$$\frac{\Delta R_6}{R_6} = B_2^p\sigma'_{11} + B_1^p\sigma'_{22} + B_3^p\sigma'_{33} - 2\sqrt{2}(B_2^p - B_3^p)\sigma'_{23} \qquad (93)$$

Superscripts n and p again denote n-type and p-type resistors, respectively. For an arbitrarily state of stress, these expressions can be inverted to solve for the six stress components in terms of the measured resistance changes

$$\sigma'_{11} = \frac{(B_3^p - B_2^p)\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_3}{R_3}\right] - (B_3^n - B_2^n)\left[\frac{\Delta R_4}{R_4} - \frac{\Delta R_6}{R_6}\right]}{2[(B_2^p - B_1^p)B_3^n + (B_1^p - B_3^p)B_2^n + (B_3^p - B_2^p)B_1^n]} + \frac{B_3^p\left[\frac{\Delta R_1}{R_1} + \frac{\Delta R_3}{R_3}\right] - B_3^n\left[\frac{\Delta R_4}{R_4} + \frac{\Delta R_6}{R_6}\right]}{2[(B_1^n + B_2^n)B_3^p - (B_1^p + B_2^p)B_3^n]}$$

$$\sigma'_{22} = -\frac{(B_3^p - B_2^p)\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_3}{R_3}\right] - (B_3^n - B_2^n)\left[\frac{\Delta R_4}{R_4} - \frac{\Delta R_6}{R_6}\right]}{2[(B_2^p - B_1^p)B_3^n + (B_1^p - B_3^p)B_2^n + (B_3^p - B_2^p)B_1^n]} + \frac{B_3^p\left[\frac{\Delta R_1}{R_1} + \frac{\Delta R_3}{R_3}\right] - B_3^n\left[\frac{\Delta R_4}{R_4} + \frac{\Delta R_6}{R_6}\right]}{2[(B_1^n + B_2^n)B_3^p - (B_1^p + B_2^p)B_3^n]}$$

$$\sigma'_{13} = \frac{\sqrt{2}}{8}\left[\frac{(B_2^p - B_1^p)\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3}\right] - (B_2^n - B_1^n)\left[\frac{\Delta R_4}{R_4} - \frac{\Delta R_5}{R_5} + \frac{\Delta R_6}{R_6}\right]}{(B_2^p - B_1^p)B_3^n + (B_1^p - B_3^p)B_2^n + (B_3^p - B_2^p)B_1^n}\right]$$

$$\sigma'_{33} = \frac{-(B_1^p+B_2^p)\left[\frac{\Delta R_1}{R_1}+\frac{\Delta R_3}{R_3}\right]+(B_1^n+B_2^n)\left[\frac{\Delta R_4}{R_4}+\frac{\Delta R_6}{R_6}\right]}{2[(B_1^n+B_2^n)B_3^p-(B_1^p+B_2^p)B_3^n]}$$

$$\sigma'_{23} = \frac{\sqrt{2}}{8}\left[\frac{-(B_2^p-B_1^p)\left[\frac{\Delta R_1}{R_1}-\frac{\Delta R_3}{R_3}\right]+(B_2^n-B_1^n)\left[\frac{\Delta R_4}{R_4}-\frac{\Delta R_6}{R_6}\right]}{(B_2^p-B_1^p)B_3^n+(B_1^p-B_3^p)B_2^n+(B_3^p-B_2^p)B_1^n}\right]$$

$$\sigma'_{12} = \frac{-(B_3^p-B_2^p)\left[\frac{\Delta R_1}{R_1}-2\frac{\Delta R_2}{R_2}+\frac{\Delta R_3}{R_3}\right]+(B_3^n-B_2^n)\left[\frac{\Delta R_4}{R_4}-2\frac{\Delta R_5}{R_5}+\frac{\Delta R_6}{R_6}\right]}{2[(B_2^p-B_1^p)B_3^n+(B_1^p-B_3^p)B_2^n+(B_3^p-B_2^p)B_1^n]} \quad (94)$$

The expressions in Eq. (94) indicate that a calibration procedure must be performed to determine all six of the combined piezoresistive parameters B_1^n, B_2^n, B_3^n, B_1^p, B_2^p, B_3^p prior to using the sensor. A combination of uniaxial and hydrostatic pressure testing can be utilized to complete this task. If the wafer is cut into specimen strips as indicated in Fig. 11 and a known uniaxial stress $\sigma'_{11}=\sigma$ is applied in the x'_1-direction, the expressions in Eq. (93) yield the following unique resistance changes

$$\frac{\Delta R_1}{R_1}=B_1^n\sigma \qquad \frac{\Delta R_3}{R_3}=B_2^n\sigma$$
$$\frac{\Delta R_4}{R_4}=B_1^p\sigma \qquad \frac{\Delta R_6}{R_6}=B_2^p\sigma \quad (95)$$

From these expressions, it is clear that the constants B_1^n, B_2^n, B_1^p, B_2^p can be easily determined using a controlled application of uniaxial stress. If the individual chips containing the sensor rosette are subjected to hydrostatic pressure ($\sigma'_{11}=\sigma'_{22}=\sigma'_{33}=-p$), the relations in Eq. (93) give

$$\frac{\Delta R_1}{R_1}=\frac{\Delta R_2}{R_2}=\frac{\Delta R_3}{R_3}=-[B_1^n+B_2^n+B_3^n]p$$
$$\frac{\Delta R_4}{R_4}=\frac{\Delta R_5}{R_5}=\frac{\Delta R_6}{R_6}=-[B_1^p+B_2^p+B_3^p]p \quad (96)$$

Therefore, the combinations $(B_1^n+B_2^n+B_3^n)$ and $(B_1^p+B_2^p+B_3^p)$ can be evaluated through controlled application of a hydrostatic pressure. The individual values of B_3^n and B_3^p can then be obtained by combining the hydrostatic pressure calibration results with the uniaxial stress calibration results.

Summary, Discussion, and Conclusions

In this paper, the theory of conduction in piezoresistive materials has been reviewed and the basic equations needed for designing test chip stress sensors have been presented. General expressions were obtained for the stress-induced resistance changes which occur in arbitrarily oriented one-dimensional filamentary conductors fabricated out of crystals with cubic symmetry and diamond lattice structure. In addition, basic equations were presented for the resistance changes experienced by stressed in-plane resistors fabricated on (100) and (111) silicon wafers.

Detailed discussion of the capabilities and limitations of several existing sensor rosettes has been given. Also, several new sensor rosettes have been presented which ease calibration considerations and permit more stress components to be measured. In particular, a pair of three element rosettes suitable for complete determination of plane stress states on (100) and (111) silicon chips have been presented. These sensor rosettes require only uniaxial loading for calibration. A new six element sensor rosette fabricated on (111) silicon was introduced which can measure the complete three-dimensional stress state at points on the surface of a die.

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